# A New class LRS Bianchi type-V string dust cosmological model in modified general relativity

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**Abstract-** Here we investigated a new class LRS Bianchi type –V string cosmological model with string as source of dust gravitational field. For the complete determination of the model, we assume that $\rho_{re} = \mu$. The physical and geometrical behaviors of cosmological model are discussed. Where $\rho_{re}$ is rest energy density and $\mu$ string tension density.

## 1. INTRODUCTION

The present universe is both spatially homogeneous and isotropic. The basic problem is cosmology to find the cosmological models of universe and to compare the resulting models with the present universe using astronomical data. In last few years of study of cosmic strings has attracted considerable interest as they believed to play an important role during early stages of the universe. Banerjee et. al. [1] have investigated an axially symmetric Bianchi Type I string dust cosmological model in presence and absence of magnetic field. Bali Upadhayay[2] have investigated LRS Bianchi –I string dust magnetized cosmological models. Pradhan et al. [3] have presented the generation of Bianchi type –V cosmological models varying ∧ term. Wang [4-6] has also discussed LRS Bianchi type I and Bianchi type III cosmological models for a cloud string with bulk viscosity. Yadav et al. [7] have studied some Bianchi type I viscous fluid string cosmological model with magnetic field. Tiwari and Sonia [8] investigated the non entence of shear in Bianchi type III string cosmological models with bulk viscosity and time dependent ∧ term. R.Bali and S. Dave [9] "Bianchi type –III string cosmological models with Bulk viscous fluid in General relativity. Bali. R. and Pareek [10] Bianchi type I string dust cosmological model with magnetic field – Recently, Baysal et. Al [11], Kilinc and Yavuz et. al.[12], Pradhan et. Al [13-14]. Singh and Tyagi [15-16] investigated various Bianchi type cosmological models with variable cosmological and gravitational constant in presence and absence of magnetic field. The perfect and bulk viscous, fluids are considered. In this paper we have investigated A new class locally rotationally symmetric (LRS) Bianchi type-V string dust cosmological model string dust as the source of gravitational field. To obtain a determine model we assume that the rest energy density is equal to the string tension density $\rho_{re} = \mu$ . The physical and geometrical behavior of cosmological model are discussed.

## 2. THE METRIC AND FIELD EQUATION

We consider the Bianchi type –V space in the metric form
$$ds^2 = -dt^2 + \alpha^2 dx^2 + \beta^2 e^{2x}(dy^2 + dz^2)$$
_____(1)
Where $\alpha$ and $\beta$ are functions of time '$t$' only.
The energy momentum tensor for string dust is given by
$$T_r^s = \rho_{re}\, v_r v^s - \mu\, x_r x^s$$
_____(2)
With
$$v_r v^r = -x^r x_r = -1 \, , v^r x_r = 0$$
_____(3)
In equation (2) $\rho_{re}$ is the rest energy density for a cloud of string with particles attached to them, $\mu$ is cloud string tension density $v^r$ is the four velocity vector of particles and $x^r$ is the unit space like vector direction of string .
In co moving co-ordinate system we get
$$v^r = (0,0,0,1); x^r = (\alpha^{-1}, 0, 0, 0)$$

_____(4)
The Physical quantities of the expansion scalar $\theta$ and shear tensor $\sigma^2$ are defined as
$$\theta = u^r; r = \frac{\dot{\alpha}}{\alpha} + \frac{2\dot{\beta}}{\beta}$$
$$= 3H$$ _____(5)

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$$\sigma^2 = \frac{1}{2}\,\sigma_{rs}\sigma^{rs}$$

$$= \frac{1}{3}\left(\theta^2 - \frac{2\dot{\alpha}\dot{\beta}}{\alpha\beta}\right.$$

$$\left. - \frac{\dot{\beta}^2}{\beta^2}\right) \underline{\hspace{4cm}}(6)$$

Where H is Hubble parameter.
The Hubble parameter is define as

$$H = \frac{\dot{\xi}}{\xi}$$

$$= \frac{1}{3}\left(\frac{\dot{\alpha}}{\alpha} + \frac{2\dot{\beta}}{\beta}\right) \underline{\hspace{4cm}}(7)$$

The average scale factor $\xi(t)$ is defined as

$$\xi(t) = (\alpha\beta^2)^{\frac{1}{3}} \underline{\hspace{4cm}}(8)$$

The Einstein's field equation with gravitational units $C = 1, 8\pi G = 1$, for a system of string

$$R_r^s - \frac{1}{2}Rg_r^s = -T_r^S \underline{\hspace{3cm}}(9)$$

For the metric (1), Einstein's field equations can be written as

$$\frac{2\ddot{\beta}}{\beta} + \frac{\dot{\beta}^2}{\beta^2} - \frac{1}{\alpha^2} = \mu$$

$$\underline{\hspace{6cm}}(10)$$

$$\frac{\ddot{\alpha}}{\alpha} + \frac{\ddot{\beta}}{\beta} + \frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} - \frac{1}{\alpha^2} = 0 \underline{\hspace{2.5cm}}(11)$$

$$\frac{2\dot{\alpha}\dot{\beta}}{\alpha\beta} + \frac{\dot{\beta}^2}{\beta^2} - \frac{3}{\alpha^2} = \rho_{re} \underline{\hspace{2.5cm}}(12)$$

$$\frac{\dot{\alpha}}{\alpha} - \frac{\dot{\beta}}{\beta} = 0 \underline{\hspace{3cm}}(13)$$

Where dot '.' On $\alpha$ and $\beta$ represent the ordinary differentiation w.r. t. $'t'$ integrating equation (13) we get

$$\log\alpha - \log\beta = \log n$$
$$\log\alpha = \log n + \log\beta$$
$$\log\alpha = \log n\beta$$

$$\alpha = n\beta$$

$$\underline{\hspace{6cm}}(14)$$

Where $n$ is the constant of integration.
From equation (14), we take n=1

$$\alpha = (1)\beta$$

$$\alpha = \beta$$

$$\underline{\hspace{6cm}}(15)$$

## 3. SOLUTION OF THE FIELD EQUATIONS:

We now obtain exact solutions of (10) – (12) which are three equations in four unknowns $\alpha, \beta, \mu$ and $\rho_{re}$ .
Therefore deterministic model, we assume a relation

$$\rho_{re} = \mu$$

$$\underline{\hspace{6cm}}(16)$$

Using the condition (16) in (10) and (12) we have

$$\frac{2\dot{\alpha}\dot{\beta}}{\alpha\beta} - \frac{2\ddot{\beta}}{\beta} - \frac{2}{\alpha^2} = 0$$

$$\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} - \frac{\ddot{\beta}}{\beta} - \frac{1}{\alpha^2}$$

$$= 0 \underline{\hspace{3.5cm}}(17)$$

Now, using equation (15) in (11) and (17) we get

$$\frac{2\ddot{\alpha}}{\alpha} + \frac{\dot{\alpha}^2}{\alpha^2} - \frac{1}{\alpha^2}$$

$$= 0 \underline{\hspace{3.5cm}}(18)$$

$$\frac{\dot{\alpha}^2}{\alpha^2} - \frac{\ddot{\alpha}}{\alpha} - \frac{1}{\alpha^2}$$

$$= 0 \underline{\hspace{3.5cm}}(19)$$

From equation (18) and (19) we get

$$\frac{2\ddot{\alpha}}{\alpha} = 0$$

$$\ddot{\alpha} = 0 \underline{\hspace{3.5cm}}(20)$$

Now integrating equation (20) we get

$$\alpha = m_1t + m_2 \underline{\hspace{3cm}}(21)$$

Where $m_1$ and $m_2$ are constant of integration.
From equation (15), we have

$$\beta = m_1t + m_2 \underline{\hspace{3cm}}(22)$$

Therefore the live element (1) can be written as

$$ds^2 = -dt^2 + (m_1t + m_2)^2 dx^2$$
$$+ (m_1t + m_2)^2 e^{2x}(dy^2$$
$$+ dz^2) \underline{\hspace{1cm}}(23)$$

## 4. SOME PHYSICAL AND GEOMETRICAL PROPERTIES

For the model equation (23), the physical and geometrical parameter can be easily obtained the rest energy density $\rho_{re}$, the string tension density $\mu$, the scalar of expansion $\theta$, Hubble parameter H, the average an isotropy parameter $A_m$ the shear scalar $\sigma$ are respectively given by

$$\xi(t)$$
$$= (m_1t + m_2) \underline{\hspace{3cm}}(24)$$

$$H = \frac{\dot{\xi}}{\xi}$$

$$= \frac{m_1}{(m_1t + m_2)} \underline{\hspace{3cm}}(25)$$

$$\theta = 3H$$

$$= \frac{3m_1}{(m_1t + m_2)} \underline{\hspace{3cm}}(26)$$

The average an isotropy parameter $A_m$ is given by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\triangle H_i}{H}\right)^2$$

Where $\triangle H_i = H_i - H \quad (i = 1,2,3)$
$$\triangle H_i = (H_1 + H_2 + H_3) - H$$
But

$$H_2 = H_3$$
$$\triangle H_i = H_1 + 2H_2 - H$$

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$$A_m = \frac{1}{3} \frac{\dfrac{4m_1^2}{(m_1 t + m_2)^2}}{\dfrac{m_1^2}{(m_1 t + m_2)^2}}$$

$$A_m = \frac{4}{3}$$

$$= \text{Constant} \underline{\hspace{4cm}} (27)$$

$$\sigma$$

$$= \frac{\sqrt{2} \, m_1}{(m_1 t + m_2)} \underline{\hspace{4cm}} (28)$$

$$\frac{\sigma}{\theta} = \frac{\sqrt{2}}{3}$$

$$= \text{Constant}$$

$$\frac{\sigma^2}{\theta^2} = \frac{2}{9}$$

$$= \text{Constant} \underline{\hspace{4cm}} (29)$$

$$\rho_{re}$$

$$= \frac{3(m_1^2 - 1)}{(m_1 t + m_2)^2} \underline{\hspace{3cm}} (30)$$

$$\mu$$

$$= \frac{3(m_1^2 - 1)}{(m_1 t + m_2)^2} \underline{\hspace{3cm}} (31)$$

## 5. DISCUSSION

1. $\frac{\sigma}{\theta} = \frac{\sqrt{2}}{3} = \text{Constant},$ Therefore model does not approach isotropy for large value of $t$.

2. As $t \to \infty$, the scale factor become infinitely large.

3. As the time $t$ increase, the rate of expansion $\theta$ decrease.

4. Average anisotropy parameter $A_m = \frac{4}{3} \neq 0$, shows early inflation and late time acceleration which is the scenanio of modern cosmology (Riess et al. 1998 and perl mutter et.al. 1999)

5. As $t \to 0$ the scalar of expansion $\theta \to \infty$ large and When $t \to \infty$, the scalar of expansion$\theta \to 0$. When at $t \to 0$, shear scalar $\sigma \to 0$. The above model describes a shearing, non ratating and expanding universe with a big-bang start.

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